1 RATIONAL NUMBERS

1. FRACTIONS

	♦	BASICS OF FRACTIONS, TYPES OF FRACTIONS AND FRACTIONS IN LOWEST TERMS
	♦	CONVERSION OF MIXED FRACTIONS INTO IMPROPER FRACTIONS AND VICE VERSA, COMPARING AND ORDERING OF FRACTIONS
	♦	HCF, LCM OF FRACTIONS AND INSERTION OF FRACTIONS BETWEEN TWO GIVEN FRACTIONS

SYNOPSIS-1

FRACTIONS

The numbers of the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$ are called fractions. Here 'a' is called the numerator and 'b' is called the denominator of the fraction $\frac{a}{b}$.

Ex. $\frac{5}{7}$ is a fraction with numerator 5 and denominator 7.

Note: i) Denominator tells us about how many equal parts the whole is divided.
ii) Numerator tells us how many parts are considered of the whole.

CLASSIFICATION OF FRACTIONS

COMMON FRACTION: The numerator is an non-negative integer and the denominator is non-zero positive integer other than 10, 100, 1000, etc., is called a vulgar fraction or common fraction or simple fraction.

Ex. $\frac{1}{8}, \frac{6}{5}$ and $\frac{10}{77}$ are some common fractions.

DECIMAL FRACTION: The denominator is the power of ten such as 10, 100, 1000, etc., in a decimal fraction.

Ex. $\frac{3}{10}, \frac{17}{100}, \frac{1}{1000}$ are some decimal fractions.

COMPLEX FRACTION: A fraction in which the numerator or denominator or both contain fractions is called a complex fraction.

Ex. $\frac{\frac{2}{7}}{3}, \frac{\frac{9}{8}}{25}$ and $\frac{\frac{7}{11}}{\frac{9}{37}}$ are some complex fractions.

PROPER FRACTIONS: Fractions in which denominator is greater than the numerator.

Ex. $\frac{2}{9}, \frac{5}{6}, \frac{2}{3}$

IMPROPER FRACTIONS: Fractions in which numerator is greater than or equal to the denominator.

Ex. $\frac{9}{2}, \frac{11}{6}, \frac{3}{2}, \frac{7}{7}$.

MIXED FRACTION: A non-negative integer together with a proper fraction is called a mixed fraction.

Ex. $3\frac{5}{7}, 8\frac{9}{25}$ and $14\frac{17}{31}$ are some mixed fractions.

Note: In $3\frac{5}{7}$, 3 is called the integral part and $\frac{5}{7}$ is called the fractional part.

Equivalent fractions: If the numerator and the denominator of a fraction are multiplied or divided by the same non-zero number, we get an equivalent fraction. The value of the fraction remains unchanged.

Ex. i) $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}, \frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}, \frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$ etc.

$\therefore \frac{2}{3} = \frac{4}{6} = \frac{10}{15} = \frac{14}{21} = ...$

ii) $\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}, \frac{18}{24} = \frac{18 \div 3}{24 \div 3} = \frac{6}{8}, \frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$

thus $\frac{18}{24} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$ are equivalent fractions.

Note: Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent, if ad = bc.

TO CONVERT A MIXED FRACTION INTO AN IMPROPER FRACTION

Multiply the integral part by the denominator and to this product, add the numerator. The result so obtained is the numerator of the required fraction.

The denominator of the required fraction will be same as the denominator of the given mixed fraction.

Thus, for given mixed fraction is $3\frac{7}{15}$, the required improper fraction

$$= \frac{(\text{integral part } \times \text{ denominator}) + \text{numerator}}{\text{Denominator}} = \frac{3 \times 15 + 7}{15} = \frac{45 + 7}{15} = \frac{52}{15}$$

Similarly, $5\frac{3}{4}=\frac{5\times4+3}{4}=\frac{20+3}{4}=\frac{23}{4}$

TO CONVERT AN IMPROPER FRACTION INTO A MIXED NUMERAL

Divide the numerator by the denominator. The quotient of this division is the integral part and the remainder obtained is numerator of the required mixed fraction.

Of course denominator will remain the same,

$$4\overline{)23}(5 \quad \underline{20} \quad 3$$

Thus, $\frac{23}{4}=\text{Quotient}\ \frac{\text{Remainder}}{\text{Denominator}}=5\frac{3}{4}$

On dividing 23 by 4, quotient = 5 and remainder = 3.

$$8\overline{)37}(4 \quad \underline{32} \quad 5$$

Similarly $\frac{37}{8}=\text{Quotient}\ \frac{\text{Remainder}}{\text{Denominator}}=4\frac{5}{8}$

SYNOPSIS-2

UNIT FRACTION: A fraction with 1 as numerator is known as unit fraction,

Numerator is one and denominator is positive integer. i.e., $\frac{1}{n}, n\in z^{+}$

Ex. $\frac{1}{1},\frac{1}{6},\frac{1}{13},\frac{1}{43}$ are unit fractions, $\frac{2}{2}$ is not an unit fraction.

Note: The reciprocal of a positive integer is an unit fraction.

SIMPLEST (LOWEST FORM OF A FRACTION: A fraction is said to be in the simplest form if its numerator and denominator have no factor in common except 1. (or) A fraction is said to be in its lowest terms or in simplest form, if the H.C.F. of its numerator and denominator is 1.

Ex. $\frac{1}{2},\frac{32}{79},\frac{99}{211},\frac{157}{371}$ are some fractions in the simplest form.

Note: A fraction in simplest form is called an irreducible fraction, otherwise it is known as a reducible fraction.

LIKE FRACTIONS: fractions with the same denominator are called like fractions.

Ex. $\frac{3}{7},\frac{1}{7},\frac{5}{7}$ are like fractions.

UNLIKE FRACTIONS: Fractions with different denominators are called unlike fractions.

Ex. $\frac{2}{11},\frac{5}{7},\frac{5}{17}$ are unlike fractions.

Ex: Which group of fractions are like fractions among the following?

i) $\frac{2}{7},\frac{3}{7},\frac{4}{7}$ ii) $\frac{1}{9},\frac{2}{9},\frac{4}{9}$ iii) $\frac{3}{7},\frac{4}{9},\frac{7}{11}$

sol. $\frac{2}{7},\frac{3}{7},\frac{4}{7}$ and $\frac{1}{9},\frac{2}{9},\frac{4}{9}$ are like fractions because having same denominators and $\frac{3}{7},\frac{4}{9},\frac{7}{11}$ are unlike fractions.

CONVERSION OF UNLIKE FRACTIONS INTO LIKE FRACTIONS

Steps 1. Find the LCM of the denominators of the fraction.
2. Divide the LCM by the respective denominators.
3. Multiply the numerator and the denominator of each fraction by the corresponding quotient obtained in step 2.

Ex. Convert $\frac{11}{16},\frac{13}{20}$ and $\frac{19}{25}$ into like fractions.

Sol. First we find the LCM of the denominators.

2	16, 20, 25
2	8, 10, 25
5	4, 5, 25
	4, 1, 5

$\therefore$ LCM $= 2\times 2\times 5\times 4\times 5 = 400$

Now $400 \div 16 = 25$, $400 \div 20 = 20$, $400 \div 25 = 16$.

$\therefore \frac{11}{16}=\frac{11\times 25}{16\times 25}=\frac{275}{400}, \frac{13}{20}=\frac{13\times 20}{20\times 20}=\frac{260}{400}$ and $\frac{19}{25}=\frac{19\times 16}{25\times 16}=\frac{304}{400}$

Hence $\frac{275}{400},\frac{260}{400}$ and $\frac{304}{400}$ are the required like fractions.

HCF OF TWO OR MORE FRACTIONS: The HCF of two or more fractions in the simplest form is the greatest fraction which exactly divides each of the given fractions.

$$\text{HCF of two or more fractions} = \frac{\text{HCF of the numerators of all the fractions}}{\text{LCM of the denominators of all the fractions}}$$

Ex. Find the HCF of $\frac{3}{8},\frac{1}{2}$ and $\frac{5}{18}$.

Sol. H.C.F of 3, 1, 5 is 1 and L.C.M of 8, 2, 18 is 72.

$$\text{HCF}=\frac{\text{HCF of 3, 1 and 5}}{\text{LCM of 8, 2 and 18}}=\frac{1}{72}$$

LCM OF TWO OR MORE FRACTIONS: The LCM of two or more fractions in the simplest form is the least fraction which is exactly divisible by each of the given fractions.

LCM of two or more fractions $= \dfrac{\text{LCM of the numerator of all the fractions}}{\text{HCF of the denominators of all the fractions}}$

Ex: Find the LCM of $\frac{3}{8}, \frac{1}{2}$ and $\frac{5}{18}$.

Sol. LCM of 3, 1, 5 is 15 and HCF of 8, 2, 18 is 2.

$$\begin{array}{r|l} 2 & 8, 2, 18 \\ \hline & 4, 1, 9 \end{array}$$

$\text{LCM} = \dfrac{\text{LCM of 3, 1 and 5}}{\text{HCF of 8, 2 and 18}} = \dfrac{15}{2}$.

INSERTION OF FRACTIONS BETWEEN TWO GIVEN FRACTIONS

A fraction between two fractions is $\dfrac{\text{sum of the numerators}}{\text{sum of the denominators}}$

i.e., A fraction between $\frac{a}{b}$ and $\frac{c}{d}$ is $\frac{a+c}{b+d}$, *i.e.*, $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$.

A fraction between $\frac{a}{b}$ and $\frac{a+c}{b+d}$ is $\frac{a+(a+c)}{b+(b+d)}$, *i.e.*, $\frac{2a+c}{2b+d}$

A fraction between $\frac{a+c}{b+d}$ and $\frac{c}{d}$ is $\frac{(a+c)+c}{(b+d)+d}$, that is $\frac{a+2c}{b+2d}$.

$\therefore \frac{a}{b} < \frac{2a+c}{2b+d} < \frac{a+c}{b+d} < \frac{a+2c}{b+2d} < \frac{c}{d}$.

Ex. A fraction between $\frac{1}{2}$ and $\frac{3}{5}$ is $\frac{1+3}{2+5}$, that is $\frac{4}{7}$.

Ex. Insert two fractions between $\frac{2}{9}$ and $\frac{3}{7}$.

Sol. One fraction between $\frac{2}{9}$ and $\frac{3}{7}$ is $\frac{2+3}{9+7} = \frac{5}{16}$. i.e., $\frac{2}{9} < \frac{5}{16} < \frac{3}{7}$.

Another fraction between $\frac{2}{9}$ and $\frac{5}{16}$ is $\frac{2+5}{9+16} = \frac{7}{25}$. i.e., $\frac{2}{9} < \frac{7}{25} < \frac{5}{16}$

$\therefore \frac{2}{9} < \frac{7}{25} < \frac{5}{16} < \frac{3}{7}$.

Note: There are an infinite number of fractions between any two fractions.

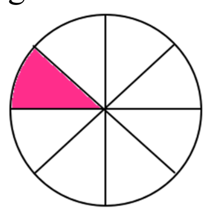
1. FRACTIONS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

BASICS OF FRACTIONS, TYPES OF FRACTIONS AND FRACTIONS IN LOWEST TERMS

1. The fraction is of the form $\frac{a}{b}$ where a, b are
   1) natural numbers 2) whole numbers
   3) integers 4) negative integers
2. In a fraction $\frac{a}{b}$, ‘a’ is called
   1) numerator 2) denominator 3) decimal 4) fraction
3. In a fraction $\frac{a}{b}$, ‘b’ is called
   1) numerator 2) denominator 3) decimal 4) fraction
4. Write the fraction representing the shaded region in the below figure.



   1) $\frac{1}{8}$ 2) $\frac{4}{8}$ 3) $\frac{5}{8}$ 4) $\frac{3}{8}$
5. A combination of a whole number and a proper fraction is called a _______ fraction.
   1) simple fraction 2) mixed fraction
   3) complex fraction 4) proper fraction
6. All mixed fractions are _______
   1) simple fractions 2) proper fractions
   3) improper fractions 4) decimal fractions
7. Which of the following are true?
   1) Fractions with same denominators are called like fractions
   2) Fractions with different denominators are called unlike fractions
   3) Both (1) and (2) 4) Neither (1) nor (2)
8. Fractions which are having one as numerator are called ______ fractions.
   1) equivalent 2) like 3) unlike 4) unit
9. The fractions in which the denominators are 10, 100, 1000, etc. are known as
   1) Decimals 2) Whole Numbers

3) Decimal fractions 4) Fractional number

10. The fractions in which the denominators are not 10, 100, 1000, etc. are known as

1) Decimal fractions 2) Decimal

3) Vulgar fractions 4) Irrational number

CONVERSION OF MIXED FRACTIONS INTO IMPROPER FRACTIONS AND VICE VERSA, COMPARING AND ORDERING OF FRACTIONS

11. The fraction $\frac{1}{12}$

1) $>\frac{7}{12}$ 2) $<\frac{7}{12}$ 3) $=\frac{8}{12}$ 4) either (1) or (2)

12. Fill in the boxes with the correct symbol : $\frac{3}{9}\boxed{\dots\dots}\frac{2}{4}$.

1) > 2) < 3) = 4) none of these

13. $\frac{20}{3}$ can be written in mixed fraction as

1) $3\frac{6}{2}$ 2) $6\frac{2}{3}$ 3) $2\frac{6}{3}$ 4) $5\frac{6}{3}$

HCF, LCM OF FRACTIONS AND INSERTION OF FRACTIONS BETWEEN TWO GIVEN FRACTIONS

14. HCF of fractions = _______

1) $\frac{\text{HCF of numerators}}{\text{HCF of denominators}}$ 2) $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

3) $\frac{\text{LCM of numerators}}{\text{LCM of denominators}}$ 4) $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

15. L.C.M of fractions = ________

1) $\frac{\text{HCF of numerators}}{\text{HCF of denominators}}$ 2) $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

3) $\frac{\text{LCM of numerators}}{\text{LCM of denominators}}$ 4) $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

16. HCF of $\frac{1}{2},\frac{2}{3}=$

1) $\frac{1}{5}$ 2) $\frac{1}{6}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

17. LCM of 2/4 and 3/4 = _____.

1) 5 2) 3/2 3) 12 4) 18

18. LCM of 5/2 and 5/3 = ______.

1) 15 2) 10 3) 5 4) 20

19. A fraction between p/q and r/s =

1) $\frac{p+r}{q+s}$ 2) $\frac{p-r}{q-s}$ 3) $\frac{q-r}{p-s}$ 4) $\frac{q+r}{p+s}$

LEVEL–II

BASICS OF FRACTIONS, TYPES OF FRACTIONS AND FRACTIONS IN LOWEST TERMS

20. The fraction with numerator is smallest 3digit number and denominator is smallest 4digit number is

1) $\frac{1}{10}$ 2) $\frac{1}{100}$ 3) 10 4) 100

21. The fraction equivalent to $\frac{5}{8}$ with denominator 32 is

1) $\frac{20}{32}$ 2) $\frac{40}{32}$ 3) $\frac{10}{32}$ 4) None

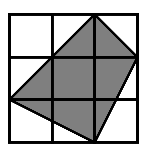
22. Simplest form of $\frac{20}{500}$ is

1) $\frac{2}{50}$ 2) $\frac{20}{50}$ 3) $\frac{1}{25}$ 4) $\frac{2}{500}$

23. Which of the following fraction is equivalent to $\frac{2}{3}$?

1) $\frac{4}{5}$ 2) $\frac{8}{6}$ 3) $\frac{10}{25}$ 4) $\frac{10}{15}$

24. What fraction of the figure is shaded?



1) $\frac{4}{9}$ 2) $\frac{1}{2}$ 3) $\frac{5}{9}$ 4) $\frac{2}{3}$

25. The next equivalent fraction of the given fraction $\frac{1}{2},\frac{2}{4},\frac{3}{6},\frac{4}{8},.....$ is

1) $\frac{7}{14}$ 2) $\frac{6}{12}$ 3) $\frac{10}{5}$ 4) $\frac{5}{10}$

26. What fraction of a day is 8 hours?

1) $\frac{1}{8}$ 2) $\frac{8}{1}$ 3) $\frac{3}{1}$ 4) $\frac{1}{3}$

27. What fraction of an hour is 45 minutes?

1) $\frac{4}{3}$ 2) $\frac{3}{4}$ 3) $\frac{1}{2}$ 4) $\frac{4}{5}$

28. The two non-zero fractions whose product is '1' are called the ___ of each other.

1) Similar 2) Proper 3) Reciprocal 4) None of these

CONVERSION OF MIXED FRACTIONS INTO IMPROPER FRACTIONS AND VICE VERSA, COMPARING AND ORDERING OF FRACTIONS

29. Which of the following statements is true?

1) $\frac{9}{16}=\frac{13}{24}$ 2) $\frac{9}{16}<\frac{13}{24}$ 3) $\frac{9}{16}>\frac{13}{24}$ 4) none of these

30. Arrange the following numbers in descending order: $-2,\frac{4}{-5},\frac{-11}{20},\frac{3}{4}$

1) $\frac{3}{4}>-2>\frac{-11}{20}>\frac{4}{-5}$ 2) $\frac{3}{4}>\frac{-11}{20}>\frac{4}{-5}>-2$

3) $\frac{3}{4}>\frac{4}{-5}>-2>\frac{-11}{20}$ 4) $\frac{3}{4}>\frac{4}{-5}>\frac{-11}{20}>-2$

31. Express $13\frac{11}{26}$ into improper fraction

1) $\frac{250}{26}$ 2) $\frac{350}{26}$ 3) $\frac{349}{26}$ 4) $\frac{351}{26}$

32. The value of $\frac{7}{30}+\frac{13}{50}+\frac{9}{5}=$

1) $\frac{192}{95}$ 2) $\frac{192}{75}$ 3) $\frac{172}{75}$ 4) $\frac{182}{75}$

HCF, LCM OF FRACTIONS AND INSERTION OF FRACTIONS BETWEEN TWO GIVEN FRACTIONS

33. HCF of $\frac{2}{3},\frac{4}{5},\frac{2}{7}$ is

1) $\frac{1}{15}$ 2) $\frac{2}{105}$ 3) $\frac{3}{5}$ 4) $\frac{3}{35}$

34. LCM of $\frac{2}{3},\frac{4}{5},\frac{3}{7}$ is

1) $\frac{12}{5}$ 2) 5 3) 12 4) 4

35. If the fraction between 3/4 and 5/6 = x/y then y = ___

1) $\frac{11x}{4}$ 2) $\frac{7x}{4}$ 3) $\frac{3x}{4}$ 4) $\frac{5x}{4}$

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

36. HCF of $\frac{2}{3},\frac{4}{5},\frac{5}{6},\frac{6}{7}$ is

1) $\frac{3}{110}$ 2) $\frac{3}{515}$ 3) $\frac{1}{210}$ 4) $\frac{1}{110}$

37. LCM of $\frac{2}{3},\frac{4}{5},\frac{5}{6},\frac{6}{7}$ is

1) 90 2) 60 3) 150 4) 200

LEVEL–IV

STATEMENT TYPE QUESTIONS

38. Statement I: HCF $\frac{3}{5},\frac{4}{25},\frac{7}{10},\frac{13}{15}=\frac{1}{150}$

Statement 2: LCM of fractions = $\frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$

1) Both statements are true. 2) Both statements are false
3) Statement I is true. Statement II is false
4) Statement I is false. Statement II is true

39. Statement I: Two or more fractions are called like fractions if they do not have the same denominator
Statement II: Fractions that have same denominator are called unlike fractions.
1) Both statements are true. 2) Both statements are false.
3) Statement I is true, statement II is false.
4) Statement I is false, statement II is true.

INTEGER TYPE QUESTIONS

40. If $7\frac{8}{9}$ is converted into improper fraction then its denominator is __________.

41. If numerators and denominators are natural numbers then HCF, LCM of fractions is always more than ______.

MULTI CORRECT ANSWER TYPE QUESTIONS

42. LCM of $\frac{2}{5},\frac{3}{7},\frac{5}{13}$ is x then x ÷ 3 is

1) 10 2) Smallest two digit whole number
3) 100 4) 30

43. Which of the following is/are equivalent to $\frac{7}{13}$?

1) $\frac{91}{65}$ 2) $\frac{63}{65}$ 3) $\frac{133}{247}$ 4) $\frac{70}{130}$

44. Which of the following is/are true

1) $\frac{3}{5}<\frac{5}{8}$ 2) $\frac{7}{10}<\frac{5}{7}$ 3) $\frac{2}{3}<\frac{7}{10}$ 4) $\frac{3}{5}<\frac{5}{7}$

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

If $\frac{a}{b},\frac{c}{d}$ are two fractions then the fraction $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$

45. The two fractions between $\frac{3}{5}$ and $\frac{5}{7}$ are

1) $\frac{3}{2},\frac{8}{5}$ 2) $\frac{3}{8},\frac{2}{5}$ 3) $\frac{2}{3},\frac{5}{8}$ 4) $\frac{1}{4},\frac{5}{3}$

46. The two fractions between $\frac{11}{16}$ and $\frac{8}{11}$ are

1) $\frac{19}{27},\frac{27}{38}$ 2) $\frac{34}{22},\frac{22}{34}$ 3) $\frac{19}{11},\frac{8}{27}$ 4) $\frac{8}{11},\frac{19}{27}$

47. The three fractions between $\frac{7}{9}$ and $\frac{13}{15}$ are

1) $\frac{33}{24},\frac{42}{34},\frac{24}{34}$ 2) $\frac{20}{24},\frac{27}{33},\frac{34}{42}$ 3) $\frac{24}{42},\frac{27}{24},\frac{33}{34}$ 4) $\frac{27}{42},\frac{20}{33},\frac{34}{24}$

MATRIX MATCH TYPE QUESTIONS

48. COLUMN-I COLUMN-II

COLUMN-I	COLUMN-II
a) 35 paise as fraction of Rs. 1	p) $\frac{4}{15}$
b) 75 cm as fraction of 2 meters	q) $\frac{2}{3}$
c) 16 hours as a fraction of 1 day	r) $\frac{7}{20}$
d) 250 gm as a fraction of 3kg	s) $\frac{3}{8}$
	t) $\frac{1}{12}$

2. DECIMALS

	♦	INTRODUCTION, DEFINITIONS AND EXPANDED FORM
	♦	CONVERSION OF A DECIMAL INTO FRACTION AND VICE VERSA
	♦	LIKE, UNLIKE DECIMALS AND COMPARISON OF DECIMALS
	♦	TERMINATING AND NON-TERMINATING REPEATING DECIMALS & VICE VERSA
	♦	CONVERSION OF NON TERMINATING REPEATING DECIMALS (PURE RECURRING DECIMAL) INTO THE FORM m/n
	♦	CONVERSION OF MIXED RECURRING DECIMAL INTO FRACTIONAL FORM

SYNOPSIS-1

INTRODUCTION

The word 'decimal' means related to ten. Thus, the fractions in which the denominators are 10, 100, 1000 etc. are known as decimal fractions.

Ex. $\frac{2}{10}, \frac{7}{100}, \frac{53}{1000}$ etc. are all decimal fractions.

1. Fractions with 10 as the denominator are called tenths.

   We know that $\frac{1}{10}$ represents the fractional number one-tenth. We also write $\frac{1}{10}$ as 0.1, read as decimal one or point one. Here the dot (.) is called the decimal point.

   We denote tenths by 1-digit after the decimal point.

2. Fractions with 100 as the denominator are called hundredths.

   We denote hundredths by 2-digits after the decimal point. We write $\frac{1}{100} = .01$.

   DECIMALS: The numbers written in decimal form are called decimal number or simple decimals.

Ex. The numbers 0.8, 0.75, 4.37 and 135.019 are decimals.

A decimal has two parts-whole number (integral) part and decimal (fractional) part.

These parts are separated by a dot (.), called the decimal part is to its right.

Ex. In 47.593, whole number part is 47 and decimal part is 593.

Note: The absence of any whole number part or decimal part of a decimal is shown by 0.

Ex. 1) .63 may be written as 0.63. 2) 89 may be written as 89.0

3) 0.74 as decimal seven four. 4) 16.08 as sixteen point zero eight.

Decimal places: The number of digits contained in the decimal part called decimal places.

Ex. 6.93 has two decimal places, those are 9 and 3 and 12.085 has three decimal places, those are 0, 8 and 5.

Like Decimals: Decimals that have same number of digits after the point (decimal) called like decimals.

Ex. 8.32, 13.78 and 120.16 are like decimals.

Unlike Decimals: Decimals having the different number of digits after the point (decimal) called unlike decimals.

Ex. 19.56, 8.137 and 0.9 are unlike decimals.

CONVERTING A DECIMAL INTO A FRACTION:

Step-1: Write the decimal without the decimal point as the numerator of the fraction.

Step-2: Write 1 followed by as many zeros as there are decimal places in the given decimal, as the denominator of the fraction.

Step-3: Simplify the fraction, if possible.

Ex. i) $0.75=\frac{75}{100}=\frac{3}{4}$ ii) $1.25=\frac{125}{100}=\frac{5}{4}=1\frac{1}{4}$ iii) $5.875=\frac{5875}{1000}=5\frac{7}{8}$

Ex: Convert decimal 12.35 into a fraction

Sol. $12.35=12+0.35=12+\frac{35}{100}=12+\frac{7}{20}=12\frac{7}{20}$

CONVERTING A FRACTION INTO A DECIMAL:

Type-1: When the denominator of a fraction is 10 or powers of 10.

Ex: Convert fraction $\frac{2125}{1000}$ into a decimal:

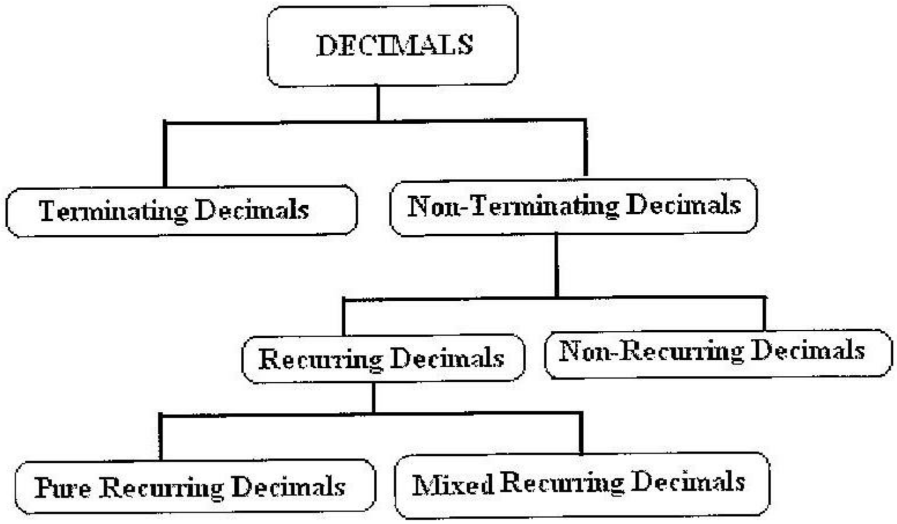
Sol. We have $\frac{2125}{1000}=\frac{2000+125}{1000}=2+\frac{125}{1000}=2+0.125=2.125$

Type-2: Fractions whose denominators are not 10 or powers of 10.

Note: Convert the denominator to 10, 100 or 1000.

Ex. i) $\frac{4}{5}=\frac{4\times 2}{5\times 2}=\frac{8}{10}=0.8$ ii) $\frac{11}{20}=\frac{11\times 5}{20\times 5}=\frac{55}{100}=0.55$

CLASSIFICATION OF DECIMALS:



TERMINATING DECIMALS:

In the process of converting a fraction into a decimal, if we obtain a zero reminder after a certain number of steps, then the decimal obtained is a terminating decimal.

Ex. $\frac{1}{5}=0.2, \frac{1}{8}=0.125, \frac{1}{25}=0.04, \frac{1}{25}=0.04, \frac{1}{4}=0.25$ and $\frac{3}{8}=0.375$ are terminating decimals.

Definition: When the fractions are expressed into decimal form, if the division process comes to an end without leaving any remainder, then the decimal is called terminating decimal.

NON-TERMINATING DECIMALS: There are situations where the division process continues indefinitely and zero remainder is never obtained. Such decimals are known as non-terminating decimals.

Ex. $\frac{1}{19}=0.5263157..., \frac{1}{17}=0.0588231...$ are non-terminating decimals.

Definition: When the fractions are expressed into decimal form, if the digit or block of digits does not repeat in the decimal, then the decimal is called non terminating non-repeating decimal.

SYNOPSIS-2

REPEATING OR RECURRING DECIMALS

Now, consider the fraction $\frac{5}{9}$.

The process of division is endless and we get a succession of 5's in the quotient.

The number 0.555… is called a recurring decimal and is written as 0.5 or $0.\overline{5}$ in short.

If in a decimal, a digit or a set of digits in the decimal part is repeated continuously, then such a number is called a recurring or repeating decimal.

Ex. $\frac{4}{9} = 0.444.... = 0.4$ and $\frac{22}{7} = 3.142857142857.... = 3.\overline{142857}$

In a recurring decimal, if a single digit is repeated then it is expressed by putting a dot on it. If a set of digits is repeated, it is expressed by putting a bar on the set.

Definition: When the fractions are expressed into decimal form, if the division process does not come to an end but a digit or block of digits repeat itself then the decimal is called non-terminating recurring decimal.

PURE RECURRING DECIMAL: A decimal in which all the digits in the decimal part are repeated, is called a pure recurring decimal.

Ex. i) $0.3, 0.\overline{43}, 0.\overline{4437}$ etc are some pure recurring decimals.

ii) $\frac{41}{333} = 0.123123...$ is written as $0.\overline{123}$.

Here, the group of digits 123 recurs repeatedly in the quotient. So, we place a dot each above the first and last digits of the group or a bar over the group.

MIXED RECURRING DECIMAL: A decimal in which some of the digits in the decimal part are repeated and the rest are not repeated, is called a mixed recurring decimal.

Ex. $\frac{7}{30} = 0.2\overline{3}, \frac{19}{30} = 0.6\overline{3}, \frac{6}{55} = 0.1\overline{09}, \frac{19}{90} = 0.2\overline{1}$ etc are mixed recurring decimals.

Note: A fraction $\frac{p}{q}$ is terminating decimal only when prime factors of q are out of 2 and 5 only.

Ex. With out division, find out, which of the following fractions are terminating decimals.

i) $\frac{5}{9}$ ii) $\frac{31}{60}$

sol. i) In $\frac{5}{9}$, we have denominator is 9 and 9 has prime factor 3 only.

$\therefore \frac{5}{9}$ cannot be expressed as a terminating decimal.

ii) In $\frac{31}{60}$, the denominator is 60 and prime factors of 60 are 2, 3 and 5.

Thus, 60 has prime factors other than 2 and 5.

$\therefore \frac{31}{60}$ cannot be expressed as a terminating decimal.

So, it is non-terminating recurring decimal.

PERIOD AND PERIODICITY OF A DECIMAL: The recurring part of the non-terminating recurring decimal is called period and the number of digits in the recurring part is called periodicity.

Ex. i) In $\frac{1}{3} = 0.\overline{3}$, Period = 3, Periodicity =1,

ii) In $\frac{3}{11} = 0.\overline{27}$, Period = 27, Periodicity = 2,

conversion of non-terminating repeating decimal number into the form of $\frac{m}{n}$:

Ex: Convert the following recurring decimals into fraction.

i) $0.\overline{3}$ ii) $0.2\overline{5}$

Sol. i) Let $x = 0.\overline{3} = 0.333....$ ...(1)

Multiplying both sides by 10, we get

$10x = 3.333......$ ...(2) ($\because$ Periodicity is 1)

Subtracting (1) from (2), we get

$10x - x = 3.333 - 0.333$

$\Rightarrow 9x = 3 \Rightarrow x = \frac{3}{9} = \frac{1}{3}$ Hence $0.\overline{3} = \frac{1}{3}$.

ii) Let $x = 0.2\overline{5} = 0.25555...$ ...(1)

multiplying both sides by 10, we get

$10x = 2.5555....$ ...(2) ($\because$ Periodicity is 1)

Subtracting (1) from (2), we get

$10x - x = 2.3 \Rightarrow x = \frac{2.3}{9} = \frac{23}{90}$. Hence $0.\overline{25} = \frac{23}{90}$

Converting a Pure Recurring Decimal into Vulgar Fraction (Short cut method):

Write the repeated digits only once in the numerator and take as many nines in the denominator as is the number of repeating digits.

Ex. i) $0.2 = \frac{2}{9}$ ii) $0.\overline{47} = \frac{47}{99}$ iii) $0.\overline{035} = \frac{35}{999}$

Converting a Mixed Recurring Decimal into Vulgar Fraction (Short cut Method)

In the numerator take the difference between the number formed by all the digits in the decimal part (taking repeated digits only once) and the number formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeroes as the number of non-repeating digits.

Ex. i) $0.21 = \frac{21-2}{99-9} = \frac{19}{90}$ ii) $0.1\overline{09} = \frac{109-1}{999-9} = \frac{108}{990} = \frac{6}{55}$

Another formula: Recurring decimal

$$= \frac{\begin{pmatrix}\text{Whole number obtained by} \\ \text{writing digits in order}\end{pmatrix} - \begin{pmatrix}\text{Whole number made by} \\ \text{non-recurring digits in order}\end{pmatrix}}{10^{(\text{Number of digits after decimal point})} - 10^{(\text{Number of digits after decimal point which does not recur})}}$$

Note: i) $a.b\overline{cd} = \frac{abcd - ab}{10^3 - 10^1}$ ii) $a.\overline{bcd} = \frac{abcd - a}{999}$

Ex. i) $7.\overline{23} = \frac{723-7}{10^2 - 10^0} = \frac{716}{99}$ ii) $32.12\overline{35} = \frac{321235-3212}{10^4 - 10^2} = \frac{318023}{9900}$

2. DECIMALS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

INTRODUCTION, DEFINITIONS AND EXPANDED FORM

1. The numbers expressed in decimal form are called
   1) decimal numbers 2) Fractions
   3) Sometimes decimal number 4) Sometimes fractional number
2. In the decimal number 4827.1454827 is called the
   1) Integral part 2) Decimal part
   3) Decimal part and Integral part 4) None
3. $20+9+\frac{4}{10}+\frac{1}{100}$ can be written in decimal as
   1) 29.04 2) 29.41 3) 2941 4) 0.2941

CONVERSION OF A DECIMAL INTO FRACTION AND VICE VERSA.

4. The fraction from 0.2
   1) $\frac{4}{5}$ 2) $\frac{3}{5}$ 3) $\frac{2}{5}$ 4) $\frac{1}{5}$
5. The decimal form of $5\frac{3}{8}$ is
   1) 5.375 2) 5.000 3) 5.255 4) 2.325
6. The number 0.125 can be written as fractions in lowest terms.
   1) $\frac{1}{8}$ 2) $\frac{125}{1000}$ 3) $\frac{25}{200}$ 4) $\frac{5}{40}$
7. Reduce 2.5 to fraction of lowest form.
   1) $\frac{25}{10}$ 2) $\frac{5}{2}$ 3) $\frac{2}{5}$ 4) $\frac{10}{25}$
8. The fractional form of the decimal number 63.7875 is:
   1) $\frac{6378}{10000}$ 2) $\frac{637}{10000}$ 3) $\frac{7875}{10000}$ 4) $\frac{637875}{10000}$

LIKE, UNLIKE DECIMALS AND COMPARISON OF DECIMALS.

9. 7.8 and 15.35 are ________ decimals.
   1) Like 2) Unlike 3) Equivalent 4) Both 1 & 2
10. Which of the following is greater?
    1) 1.09 2) 0.19 3) 1.90 4) 1.009
11. Which of the following is smaller?
    1) 0.7 2) 0.07 3) 0.007 4) 0.0007
12. Which of the following is true
    1) $0.3 > 0.4$ 2) $0.07 < 0.02$ 3) $3 > 0.8$ 4) $0.5 = 0.05$

TERMINATING AND NON-TERMINATING REPEATING DECIMALS & VICE VERSA

13. The recurring part of the non - terminating recurring decimal is called ______
    1) Period 2) Periodicity
    3) Both (1) & (2) 4) Neither (1) nor (2)

14. The number of digits in the recurring part of the non - terminating recurring decimal is called________.
    1) Period 2) Periodicity 3) Both (1) & (2) 4) Neither (1) nor (2)

15. In $0.\overline{26}$ period is
    1) 2 2) 6 3) 26 4) 62

16. In $0.\overline{46}$ periodicity is
    1) 4 2) 6 3) 46 4) 2

17. While converting the fractional number into decimal the process of division terminates, then the decimal is _____
    1) Terminating 2) Non-terminating
    3) Repeating 4) Non-terminating&non-repeating

CONVERSION OF NON TERMINATING REPEATING DECIMALS (PURE RECURRING DECIMAL) INTO THE FORM m/n

18. 1.272727……… can be expressed in fractional form as
    1) $\frac{14}{99}$ 2) $\frac{14}{11}$ 3) $\frac{11}{14}$ 4) $\frac{99}{14}$

19. Express the following recurring decimal expansions in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. $0.\overline{4}$.
    1) $\frac{1}{9}$ 2) $\frac{2}{9}$ 3) $\frac{4}{9}$ 4) $\frac{5}{9}$

CONVERSION OF MIXED RECURRING DECIMAL INTO FRACTIONAL FORM

20. $0.3\overline{07} =$
    1) $\frac{152}{495}$ 2) $\frac{495}{152}$ 3) $\frac{475}{150}$ 4) $\frac{150}{475}$

21. The value of $0.4\overline{7}$
    1) $\frac{23}{60}$ 2) $\frac{43}{90}$ 3) $\frac{43}{50}$ 4) $\frac{73}{90}$

LEVEL–II

INTRODUCTION, DEFINITIONS AND EXPANDED FORM

22. 3 hundredths can be written as
1) 0.003 2) 0.03 3) 0.300 4) 300

23. Three hundred six and seven hundredth in decimal form can be written as
1) 306700 2) 306.7 3) 306.07 4) 30670

24. Which of the following number can be placed in the tens column if the given number is 297.35
1) 2 2) 9 3) 7 4) 3

25. Which of the following number can be placed in the ones column if the given number is 97.50
1) 9 2) 5 3) 7 4) 0

26. The place value of '5' in 987.352
1) 0.35 2) 0.05 3) 0.5 4) 5

CONVERSION OF A DECIMAL INTO FRACTION AND VICE VERSA.

27. Which is the fraction not equivalent to 1.670?
1) $\frac{167}{100}$ 2) $\frac{1670}{1000}$ 3) $\frac{16.7}{10}$ 4) 16.7

28. 0.06 = ?
1) $\frac{3}{5}$ 2) $\frac{3}{50}$ 3) $\frac{3}{500}$ 4) none of these

29. 1.04 = ?
1) $1\frac{1}{5}$ 2) $1\frac{2}{5}$ 3) $1\frac{1}{25}$ 4) none of these

LIKE, UNLIKE DECIMALS AND COMPARISON OF DECIMALS.

30. If $A=5.39, B=5.039$ and $C=5.9304$ then which of the following is true?
1) C>A 2) B<A 3) B<C 4) All of these

31. Which is greater among 2.3, 2.03, 2.33, 2.05?
1) 2.3 2) 2.03 3) 2.33 4) 2.05

32. 348.111, 432.89, 999.9, 26.3512 are
1) Like 2) Unlike 3) Conversion 4) None

33. Write the following decimals in descending order: 9.03, 4.85, 0.974, 7.5, 4.92 and 0.7.
1) 9.03, 7.5, 4.92, 4.85, 0.974, 0.7 2) 7.5, 9.03, 4.85, 4.92, 0.7, 0.974
3) 0.7, 7.5, 9.03, 4.85, 4.92, 0.974 4) None of these

34. Write the following decimals in ascending order: 6.7, 7.3, 3.7, 6.07, 7.37, 3.71
1) 3.71, 6.07, 3.7, 7.3, 6.7, 7.37 2) 3.7, 3.71, 6.07, 6.7, 7.3, 7.37
3) 7.37, 3.71, 3.7, 6.07, 7.3, 6.7, 4) None

TERMINATING AND NON-TERMINATING REPEATING DECIMALS& VICE VERSA

35. The period of the decimal form of $\frac{22}{7}$ is
1) 6 2) 5 3) 142857 4) 14857

36. A non terminating decimal from the following is
1) $\frac{3}{16}$ 2) $\frac{5}{125}$ 3) $\frac{9}{40}$ 4) $\frac{7}{27}$

37. A terminating decimal from following is
1) $\frac{3}{16}$ 2) $\frac{1}{15}$ 3) $\frac{5}{14}$ 4) $\frac{2}{11}$

38. Which of the following statement is true?
1) $\frac{1}{22}$ can be written as a terminating decimal.
2) $3\frac{5}{7}$ can be written as a terminating decimal.
3) $\frac{2}{15}$ can be written as a terminating decimal.
4) $\frac{1}{50}$ can be written as a terminating decimal.

39. Express the rational number $\frac{1}{13}$ in decimal form and hence find the decimal expansion of $4\frac{4}{13}$
1) $5.\overline{207692}$ 2) $11.\overline{307692}$ 3) $4.\overline{307692}$ 4) $6.\overline{308692}$

CONVERSION OF NON TERMINATING REPEATING DECIMALS (PURE RECURRING DECIMAL) INTO THE FORM m/n

40. Express the following recurring decimal expansions in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. $2.\overline{124}$
1) $\frac{2122}{333}$ 2) $\frac{2122}{666}$ 3) $\frac{2122}{999}$ 4) $\frac{2122}{222}$

41. The value of $0.\overline{09} =$
1) $\frac{9}{100}$ 2) $\frac{1}{11}$ 3) $\frac{9}{11}$ 4) $\frac{1}{99}$

CONVERSION OF MIXED RECURRING DECIMAL INTO (m/n FORM)

42. The value of $0.23\overline{7}$
1) $\frac{47}{198}$ 2) $\frac{57}{189}$ 3) $\frac{57}{198}$ 4) $\frac{75}{198}$

43. $0.12\overline{3}$ can be expressed in fractional form as

1) $\frac{900}{111}$ 2) $\frac{111}{900}$ 3) $\frac{123}{10}$ 4) $\frac{121}{900}$

44. Express $0.2\overline{35}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

1) $\frac{433}{890}$ 2) $\frac{132}{900}$ 3) $\frac{233}{990}$ 4) $\frac{333}{690}$

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

45. Find the value of $2.\overline{6} - 0.\overline{9}$

1) $\frac{5}{3}$ 2) $\frac{1}{3}$ 3) $\frac{7}{2}$ 4) $\frac{1}{2}$

46. Simplify $0.88\overline{5} - 0.3\overline{53}$

1) $0.63\overline{10}$ 2) $0.43\overline{30}$ 3) $0.53\overline{10}$ 4) $0.53\overline{20}$

LEVEL–IV

STATEMENT TYPE QUESTIONS

47. Statement I: $\frac{9}{11}$ is a pure recurring decimal.

Statement II: A non-terminating recurring decimal which has no repeating digit after the decimal point Immediately is called pure recurring decimal.

1) Both the statements are true 2) Both the statements are false

3) Statement I is true, statement II is false

4) Statement I is false, statement II is true.

INTEGER TYPE QUESTIONS

48. The length of the period of the decimal form of $\frac{289}{13}$ is __________6

49. In $0.\overline{3}$ period is ______

MULTI CORRECT ANSWER TYPE QUESTIONS

50. A non-terminating decimals form the following are

1) $\frac{3}{11}$ 2) $\frac{81}{64}$ 3) $\frac{9}{100}$ 4) $\frac{11}{17}$

51. When 0.125125…. is converted into fraction the result is

1) $\frac{125}{999}$ 2) $\frac{250}{990}$ 3) $\frac{500}{1980}$ 4) $\frac{250}{1998}$

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

Every fractional number can be expressed in decimal form is expressible in terminating (or) repeating decimals form.

52. Which of the following is expressed as repeating decimal?

1) $\frac{25}{400}$ 2) $\frac{22}{25}$ 3) $\frac{20}{7}$ 4) $\frac{15}{40}$

53. Which of the following is expressed as terminating decimal?

1) $\frac{11}{13}$ 2) $\frac{14}{17}$ 3) $\frac{81}{31}$ 4) $\frac{251}{400}$

54. The decimal form of $\frac{21}{5}$

1) 4.2 2) 4.02 3) 4.002 4) 4.0002

MATRIX MATCH TYPE QUESTIONS

55.

COLUMN-I	COLUMN-II
a) $\frac{201}{202}$ expressed as a	p) Terminating decimal
b) $\frac{201}{256}$ expressed as a	q) $\frac{402}{458}$
c) The number which is lies between $\frac{201}{202}$ and $\frac{201}{256}$ are	r) Repeating decimal
d) $\left(\frac{p}{q}\times\frac{m}{n}\right)\times\frac{x}{y}=\frac{p}{q}\left(\frac{m}{n}\times\frac{x}{y}\right)$ is a	s) Non-terminating & non-repeating
	t) Associative law of multiplication

3. INTRODUCTION TO RATIONAL NUMBERS

	♦	INTRODUCTION TO RATIONAL NUMBERS
	♦	REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE AND RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

SYNOPSIS-1

RATIONAL NUMBERS

INTRODUCTION: So far we have discussed fractions, decimals and the basic arithmetic operations on these numbers. Sometimes we face certain situations, where the understanding of numbers we have already studied may not be sufficient.

Ex. Temperature at Jammu on a certain day is $5\frac{1}{2}$ below zero degrees. i.e., $-5\frac{1}{2}$.

This number is neither an integer nor a fraction. It is a rational number.

Definition: A number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number. The set of rational numbers is denoted by the capital letter 'Q', and Q comes from the word 'Quotient'.

Ex. $0, 1, \frac{3}{4}, \frac{5}{6}, \frac{-2}{3}, \frac{6}{7}, \ldots.$

Note: The word 'rational' is derived from the word 'ratio'.

DIFFERENCE BETWEEN A FRACTION AND A RATIONAL NUMBER:

Fraction	Rational Number
A number of the form $\frac{p}{q}$, where p and q are whole numbers and $q \neq 0$ is called fraction.	A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number

POSITIVE RATIONAL NUMBER: If $\frac{p}{q}$ is a rational number and $\frac{p}{q} > 0$, then $\frac{p}{q}$ is called positive rational number. (or)

A rational number is said to be positive if its numerator and denominator are either both positive or both negative.

Ex. 1) Set of all positive integers 2) Set of all natural numbers

3) Set of all fractions

NEGATIVE RATIONAL NUMBER: If $\frac{p}{q}$ is rational number and $\frac{p}{q} < 0$, then $\frac{p}{q}$ is called an negative rational number. (or)

A rational number is said to be negative if either the numerator or the denominator is negative.

Ex. i) Set of negative integers ii) Set of negative of the fractions

Note: i) '0' is neither positive nor negative but, it is rational.

ii) A rational number $\frac{p}{-q}$ is expressed as $\frac{-p}{q}$.

iii) All natural numbers, whole numbers and integers are rational numbers, but all rational numbers are need not be integers, whole numbers and natural numbers.

Ex. $-3, -2, -1, 0, 1, 2, 3, \ldots$ are rationals.

iv) All fractions are rational numbers, but every rational number need not be a fraction.

Ex. (1) $\frac{1}{2}, \frac{3}{4}, \frac{1}{6}, \ldots$ are fractions as well as rational numbers.

(2) $2, 3, \frac{-5}{2}, \ldots$ are rational numbers but not fractions.

v) Decimal numbers that can be written in the form of $\frac{p}{q}$ are rational numbers.

Ex. $0.125 = \frac{1}{8}, 6.25 = \frac{25}{4}, -2.5 = -\frac{5}{2}$

vi) All numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ can be expressed as either terminating decimals or non-terminating recurring decimals. So, every terminating or recurring decimal is a rational number.

Ex. $\frac{25}{4} = 6.25, \frac{1}{3} = 0.33333\ldots$ are rational numbers.

EQUIVALENT RATIONAL NUMBERS : The equality of two rational numbers can be checked by any one of the following methods.

Method-I: If the numerator and denominator of a given rational number are multiplied (or divided) by the same non-zero integer, then the new rational number thus formed is said to be equivalent to the given rational number.

i.e., Rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are equivalent to each other if $\frac{p\times k}{q\times k}=\frac{r}{s}$ or $\frac{p\div k}{q\div k}=\frac{r}{s}$ [k is a non-zero integer]

Ex. $\frac{48}{52}=\frac{12}{13}$ and $\frac{12\times 4}{13\times 4}=\frac{48}{52}$

Method-2: If the product of the numerator of the 1st rational number and the denominator of the 2nd rational number is equal to the product of the numerator of the 2nd rational number and the denominator of the 1st rational number then they are known as equivalent rational numbers.

i.e., Rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent to each other if $p\times s=r\times q$

Ex. (1) $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent to each other because $2\times 9=6\times 3$

(2) $\frac{3}{4}=\frac{15}{20}\Rightarrow 3\times 20=15\times 4\Rightarrow 60=60$

Note: A rational number has infinite equivalent rational numbers.

SYNOPSIS-2

REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

Step-1: Draw a line and mark a point 'O' on it to represent 9 (zero).

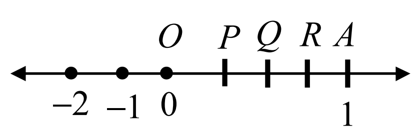
Step-2: Mark points at equal distance on the left and right side of 0, such that distance between two adjacent points is 1 unit.

Step-3: Then divide the unit into equal parts such that distance between the parts is equal to the denominator of rational number.

Step-4: The numerator tells how many of these parts are considered.

Ex. To represent $\frac{3}{4}$ on the number line first we draw a number line.

Let 'O' represents 0(zero) and 'A' represents 1. So divide OA into 4 equal periods label each point as P,Q and R. Point R represents $\frac{3}{4}$.



Ex: Represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line.

Sol. $\frac{5}{3}$ and $\frac{-5}{3}$ can be written as $1\frac{2}{3}$ and $-1\frac{2}{3}$.

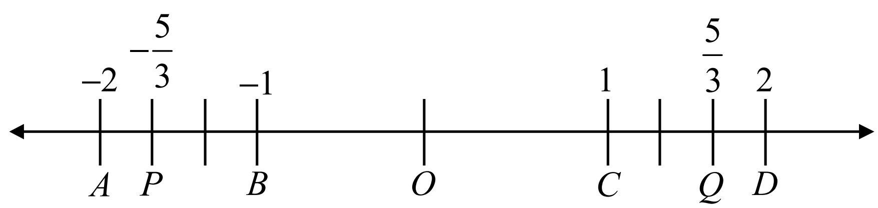
Step-1: In order to represent $\frac{5}{3}$ and $\frac{-5}{3}$ on he umber line, we draw a number line and mark a point O on it to represent zero.

Step-2: Since $1\frac{2}{3}$ and $-1\frac{2}{3}$ lie between 1 & 2 and –1 & –2 therefore mark the points A and B & C and D on left & right side of O, such that A, B, C, D represent –2, –1, 1, 2.

Step-3: Now the denominator of rational number is 3.

$\therefore$ Divide the intervals into 3 equal parts.

Step-4: Since the numerator is 2, then mark second point on the parts. P and Q are the required points.



COMPARISON OF RATIONAL NUMBERS: Let us understand two cases in comparison of Rational numbers. For that process, let us keep the following points in mind.

1. A positive rational number is always greater than a negative rational number
2. Zero is greater than each one of the negative rational numbers and less than each one of the positive rational numbers.
3. Rational numbers can be divided into two types.
   i) Rational numbers having the same denominator.
   ii) Rational numbers having the different denominators.

Comparison of Rational Numbers when the denominators are equal:

If the denominators of two given rational numbers are equal, then by comparing the numerators we can compare those rational numbers.

Ex. $\frac{-5}{7}$ and $\frac{-2}{7}$

Here, denominators are 7, 7 which are equal and numerators are –5, –2.

Since denominators are equal, let us compare the numerators.

We know that $-5 < -2$. Hence $\frac{-5}{7} < \frac{-2}{7}$

Comparison of Rational Numbers when the denominators are not equal:

If the denominators of two rational numbers are not equal, then we should change them into fractions having the same denominators.

We can achieve in three steps.

Step-1: Multiplying numerator and denominator of first rational number by the denominator of second rational number and

Step-2: Multiplying numerator and denominator of second rational number by the denominator of first rational number.

Step-3: Once the fractions have equal denominators, compare their numerators.

Ex. $\frac{3}{5}$ and $\frac{4}{7}$

Here, denominators are 5,7 which are not equal, and numerators are 3,4.

Since denominators are not equal, we have to change the given fractions into fractions having the same denominators.

$\frac{3\times 7}{5\times 7} = \frac{21}{35}$ and $\frac{4\times 5}{7\times 5} = \frac{20}{35}$

Now denominators of $\frac{21}{35}$ and $\frac{20}{35}$ are equal and $21 > 20$

$\therefore \frac{21}{35} > \frac{20}{35}$. *i.e.,* $\frac{3}{5} > \frac{4}{7}$

Ex: Compare $\frac{-8}{9}$ and $\frac{-4}{5}$.

Sol. L.C.M. of 9 and 5 is $9 \times 5 = 45$.

$\frac{-8}{9} = \frac{-8\times 5}{9\times 5} = \frac{-40}{45}, \frac{-4}{5} = \frac{-4\times 9}{5\times 9} = \frac{-36}{45}$

Since $-40 < -36$

$\frac{-40}{45} < \frac{-36}{45}$

$\therefore \frac{-8}{9} < \frac{-4}{5}$

SYNOPSIS-3

DENSITY PROPERTY OF RATIONAL NUMBERS

Between any two given rational numbers there exists uncountable rational numbers. This property of rational numbers is called the property of density.

Theorem: Show that between any two distinct rational numbers a and b, there exists another rational number.

Sol. Since $a \neq b$, without any loss of generality we may assume that $a < b$.

Now $a < b$,

$$\therefore a + a < a + b \Rightarrow 2a < a + b \Rightarrow a < \frac{a+b}{2} \qquad .....(1)$$

Also, $a < b$

$$\therefore a + b < b + b \Rightarrow a + b < 2b \Rightarrow \frac{a+b}{2} < b \qquad .....(2)$$

From (1) & (2), $a < \frac{a+b}{2} < b$

Clearly, $\frac{a+b}{2}$ is a rational number lying between a and b.

Note: i) A rational number between any two rational numbers a and b is $\frac{1}{2}(a+b)$.

Ex. A rational number between $\frac{1}{4}$ and $\frac{2}{3}$ is

$$\frac{1}{2}\left(\frac{1}{4}+\frac{2}{3}\right) = \frac{1}{2}\left(\frac{3+8}{12}\right) = \frac{1}{2}\left(\frac{3+8}{12}\right) = \frac{1}{2} \times \frac{11}{12} = \frac{11}{24}$$

ii) Let a and b be two rational numbers such that $a < b$. Suppose to find 'n' rational numbers between a and b. Let $d = \frac{b-a}{n+1}$. Then 'n' rational numbers lying between a and b are $(a+d),(a+2d),(a+3d),...(a+nd)$.

Ex. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. Let five rational numbers a, b, c, d, e are in between $\frac{3}{5}$ and $\frac{4}{5}$

i.e., $\frac{3}{5} < d < b < a < c < e < \frac{4}{5}$

$a = \frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right) = \frac{7}{10}$, $\qquad b = \frac{1}{2}\left(\frac{3}{5}+\frac{7}{10}\right) = \frac{13}{20}$

$c = \frac{1}{2}\left(\frac{7}{10}+\frac{4}{5}\right) = \frac{15}{20} = \frac{3}{4}$ $d = \frac{1}{2}\left(\frac{3}{5}+\frac{13}{20}\right) = \frac{25}{40} = \frac{5}{8}$ $e = \frac{1}{2}\left(\frac{3}{4}+\frac{4}{5}\right) = \frac{31}{40}$

i.e., $\frac{3}{5} < \frac{5}{8} < \frac{13}{20} < \frac{7}{10} < \frac{3}{4} < \frac{31}{40} < \frac{4}{5}$.

Hence, the required rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{5}{8}, \frac{13}{20}, \frac{7}{10}, \frac{3}{4}, \frac{31}{40}$

Ex. Find 8 rational numbers between 2 and 3.

Sol. Since we want 8 rational numbers between 2 and 3.

So, we convert 2 and 3 with denominator $8+1$ i.e., 9.

$2 = \frac{2\times 9}{9} = \frac{18}{9}$ and $3 = \frac{3\times 9}{9} = \frac{27}{9}$

Now, we have to insert 8 rational numbers between $\frac{18}{9}$ and $\frac{27}{9}$.

On increasing numerators by 1 of $\frac{18}{9}$ successively, we get the required 8 numbers $\frac{19}{9}, \frac{20}{9}, \frac{21}{9}, \frac{22}{9}, \frac{23}{9}, \frac{24}{9}, \frac{25}{9}, \frac{26}{9}$.

Ex. Find seven rational numbers between $\frac{1}{6}$ and $\frac{5}{21}$

Sol. We have $\frac{1}{6}$ and $\frac{5}{21}$. L.C.M of 6 and 21 is 42.

To make denominators equal i.e., 42, we multiply numerator and denominator of $\frac{1}{6}$ by 7 and that of $\frac{5}{21}$ by 2, we get, $\frac{7}{42}$ and $\frac{10}{42}$.

To insert seven rational numbers we multiply the numerators and denominators of both the fractions by such a number 2 or 3 or 4……, so that the difference between the numerators is at least 7.

Multiplying the numerators and denominators of both fractions by 3, we get,

$\frac{7}{42}\times\frac{3}{3} = \frac{21}{126}$ and $\frac{10}{42}\times\frac{3}{3} = \frac{30}{126}$

On increasing the numerators by 1 of $\frac{21}{126}$ successively, we get the required seven rational numbers between $\frac{1}{6}$ and $\frac{5}{21}$ are

$\frac{22}{126}, \frac{23}{126}, \frac{24}{126}, \frac{25}{126}, \frac{26}{126}, \frac{27}{126}, \frac{28}{126}$.

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

INTRODUCTION TO RATIONAL NUMBERS

1. If a, b are two whole numbers and $b \neq 0$ then $\frac{a}{b}$ said to be always_____

   1) Integer 2) Natural number
   3) whole number 4) Fraction

2. A number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a _______.

   1) Rational number 2) Natural number
   3) whole number 4) Fraction

3. Which of the following statement is False?

   1) Every fraction is a rational number
   2) Every rational number is a fraction
   3) Every integer is a rational number
   4) All of these

4. Which of the following is an greatest rational numbers $\frac{-11}{28}, \frac{-5}{7}, \frac{9}{-14}, \frac{29}{-42}$?

   1) $\frac{-11}{28}$ 2) $\frac{-5}{7}$ 3) $\frac{9}{-14}$ 4) $\frac{29}{-42}$

5. Which of the following forms a pair of equivalent rational numbers?

   1) $\frac{14}{35}$ *and* $\frac{21}{45}$ 2) $\frac{-12}{26}$ *and* $\frac{-18}{39}$ 3) $\frac{-7}{28}$ *and* $\frac{-5}{20}$ 4) Both (2) and (3)

6. The reciprocal of a negative rational number

   1) is a positive rational number
   2) is a negative rational number
   3) can be either a positive or a negative rational number
   4) its not rational number

7. Which of the following rational numbers is in the standard form?

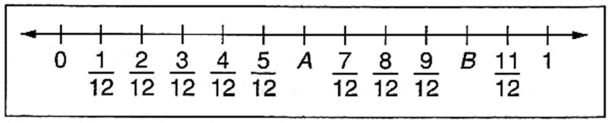
   1) $\frac{-12}{26}$ 2) $\frac{-49}{91}$ 3) $\frac{-9}{16}$ 4) $\frac{-6}{15}$

8. The standard form of $\frac{192}{-168}$ is

   1) $\frac{-2}{3}$ 2) $\frac{-8}{7}$ 3) $\frac{-1}{7}$ 4) $\frac{-6}{7}$

REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE AND RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

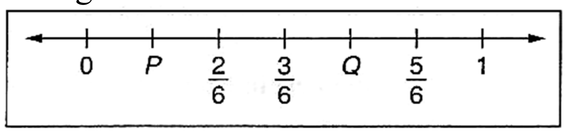
9. Consider the following number line



What is the value of A+B?

1) $\frac{4}{3}$ 2) $\frac{3}{4}$ 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

10. Consider the following number line



What is the product of reciprocals of P and Q?

1) 9 2) 6 3) 3 4) 2

11. Number of rational numbers are there between any two rational numbers?

1) 0 2) 3 3) 4 4) Infinite

12. A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ is

1) $\frac{7}{24}$ 2) 0.29 3) $\frac{13}{48}$ 4) All of these

LEVEL–II

INTRODUCTION TO RATIONAL NUMBERS

13. Which of the following statements is True?

1) $\frac{5}{7}<\frac{7}{9}<\frac{9}{11}<\frac{11}{13}$ 2) $\frac{11}{13}<\frac{9}{11}<\frac{7}{9}<\frac{5}{7}$ 3) $\frac{5}{7}<\frac{11}{13}<\frac{7}{9}<\frac{9}{11}$ 4) $\frac{5}{7}<\frac{9}{11}<\frac{11}{13}<\frac{7}{9}$

14. The value of x for which the two rational numbers $\frac{3}{7},\frac{x}{42}$ are equivalent, is

1) 18 2) 15 3) 12 4) 10

REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE AND RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

15. The following are any three rational numbers between 3 and 4

1) $\frac{7}{3},\frac{13}{5},\frac{15}{2}$ 2) $\frac{7}{4},\frac{15}{2},\frac{13}{7}$ 3) $\frac{7}{2},\frac{15}{4},\frac{13}{4}$ 4) None of these

16. The three rational numbers between -2 and 5 are

1) $\frac{-1}{3},\frac{2}{3},\frac{11}{4}$ 2) $\frac{-1}{4},\frac{3}{2},\frac{13}{4}$ 3) $\frac{-1}{5},\frac{2}{5},\frac{11}{5}$ 4) All of these

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

17. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$

1) $\frac{97}{160}<\frac{98}{160}<\frac{99}{160}<\frac{103}{160}<\frac{101}{160}<\frac{102}{160}<\frac{107}{160}<\frac{108}{160}<\frac{110}{160}<\frac{106}{160}$
2) $\frac{97}{160}<\frac{98}{160}<\frac{99}{160}<\frac{100}{160}<\frac{101}{150}<\frac{102}{150}<\frac{103}{150}<\frac{104}{150}<\frac{105}{150}<\frac{106}{160}$
3) $\frac{96}{160}<\frac{98}{160}<\frac{99}{160}<\frac{100}{160}<\frac{110}{160}<\frac{111}{160}<\frac{112}{160}<\frac{114}{160}<\frac{115}{160}<\frac{116}{160}$
4) $\frac{97}{160}<\frac{98}{160}<\frac{99}{160}<\frac{100}{160}<\frac{101}{160}<\frac{102}{160}<\frac{103}{160}<\frac{104}{160}<\frac{105}{160}<\frac{106}{160}$

LEVEL–IV

STATEMENT TYPE QUESTIONS

18. Statement I: One of the rational between $\frac{1}{5}$ and $\frac{1}{4}$ is $\frac{9}{40}$

Statement II: If x and y are any two rational numbers such that x < y, then $\frac{1}{2}(x+y)$ is a rational number between x and y such that $x<\frac{1}{2}(x+y)<y$

1) Both Statements are true. 2) Both Statements are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

19. Statement – I: In a rational number denominator always has to be a non-zero integer

Statement – II: Every fraction is a rational number.

1) Both the statements are true 2) Both the statements are false
3) Statement I is true. Statement II is false
4) Statement I is false. Statement II is true.

20. Statement – I: Two rational numbers with different numerators can never be equals.

Statement – II: The rational number $\frac{-2}{3}$ lies to the right of zero on the number line.

1) Both the statements are true 2) Both the statements are false
3) Statement I is true. Statement II is false
4) Statement I is false. Statement II is true.

INTEGER TYPE QUESTIONS

21. In every Rational number denominator should not be _______.

22. The number of rational numbers whose absolute value is zero____

MULTI CORRECT ANSWER TYPE QUESTIONS

23. Which of the following rational numbers has their standard form of $\frac{3}{5}$

1) $\frac{-6}{15}$ 2) $\frac{9}{-15}$ 3) $\frac{-9}{-15}$ 4) $\frac{15}{25}$

24. Which of the following are not false?

1) Every positive Rational number is greater than zero
2) Every negative Rational number is less than zero
3) Every Rational number can be represented on a number line
4) 0 is a rational number its reciprocal is not defined

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

A rational number line is one whose points are associated with rational numbers. Each point on this line is associated with a rational number one each number is associated with a point on the line.

25. The rational numbers $\frac{1}{16}$ and -1 are on opposite sides of zero on the number line.

Is this statement

1) True 2) False
3) Neither true nor false 4) None of these

26. How many rational numbers are there between any two rational numbers on the number line.

1) 1 2) 0 3) Unlimited 4) 100

27. The number line extends ___ on both the sides

1) Unlimited 2) +100 and –100 3) $+\frac{1}{100}$ and $-\frac{1}{100}$ 4) None of these

MATRIX MATCH TYPE QUESTIONS

28. Match each item of Column I with the corresponding item of Column II.

COLUMN I	COLUMN II
a) A positive rational number	p) $-\frac{2}{3}$
b) A negative rational number	q) $\frac{9}{12}$
c) The standard form of $\frac{8}{-12}$	r) $-\left(\frac{-7}{3}\right)$
d) A rational number equivalent to $\frac{3}{4}$	s) $\frac{10}{-36}$

4. OPERATIONS AND LAWS OF RATIONAL NUMBERS

	♦	OPERATIONS ON RATIONAL NUMBERS
	♦	LAWS OF RATIONAL NUMBERS

SYNOPSIS-1

OPERATIONS ON RATIONAL NUMBERS

ADDITION OF RATIONAL NUMBERS:

1. In order to add two rational numbers

i) First express each rational number with positive denominator.

ii) If the denominators of both the rational numbers are same, then add their numerators and divide by the common denominator.

iii) If the denominator of both the rational numbers are different, then we express them as equivalent rational numbers with the same denominator then add them.

Ex: Add $\frac{4}{-11}$ and $\frac{7}{11}$

Sol. $\frac{4}{-11}+\frac{7}{11}=\frac{-4}{11}+\frac{7}{11}=\frac{-4+7}{11}=\frac{3}{11}$

Ex: Find the sum of the rational numbers $\frac{-4}{9},\frac{15}{12}$ and $\frac{-7}{18}$.

Sol. $\frac{-4}{9}+\frac{15}{12}+\frac{-7}{18}=\frac{-16+45-14}{36}=\frac{15}{36}=\frac{5}{12}$

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Closure Property: Let $\frac{a}{b}$ and $\frac{c}{d}$ are two unique rational numbers such that $\frac{a}{b}+\frac{c}{d}$ is also a rational number.

Ex. Let $\frac{-5}{12},\frac{-1}{4}$ are any two rational numbers, $\left(\frac{-5}{12}+\frac{-1}{4}\right)=\frac{-5-3}{12}=\frac{-8}{12}=\frac{-2}{3}$, here $\frac{-2}{3}$ is also a rational number.

COMMUTATIVE PROPERTY: Let $\frac{a}{b}$ and $\frac{c}{d}$ are two unique rational numbers such that $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$.

Ex. $\frac{3}{5}+\frac{11}{17}=\frac{51+55}{85}=\frac{106}{85}$ and $\frac{11}{17}+\frac{3}{5}=\frac{55+51}{85}=\frac{106}{85}$.

$\therefore \frac{3}{5}+\frac{11}{17}=\frac{11}{17}+\frac{3}{5}$

ASSOCIATIVE PROPERTY: Let $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers such that

$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$.

Ex. Consider three rational numbers $\frac{1}{2},\frac{3}{7}$ and $\frac{5}{9}$.

Let $\left(\frac{1}{2}+\frac{3}{7}\right)+\frac{5}{9}=\left(\frac{7+6}{14}\right)+\frac{5}{9}=\frac{13}{14}+\frac{5}{9}=\frac{117+70}{126}=\frac{187}{126}$

And $\frac{1}{2}+\left(\frac{3}{7}+\frac{5}{9}\right)=\frac{1}{2}+\frac{62}{63}=\frac{63+124}{126}=\frac{187}{126}$

$\therefore \left(\frac{1}{2}+\frac{3}{7}\right)+\frac{5}{9}=\frac{1}{2}+\left(\frac{3}{7}+\frac{5}{9}\right)$

ADDITIVE IDENTITY (RULE OF 0) :

$a+0=0+a=a$; '0' is called the additive identity of 'a'.

Ex. '0' is a rational number such that the sum of any rational number and 0 is the rational number itself.

$\frac{7}{9}+0=\frac{7}{9}, 0+\frac{7}{9}=\frac{7}{9}$ $\therefore \frac{7}{9}+0=0+\frac{7}{9}=\frac{7}{9}$

ADDITIVE INVERSE:

Negative or additive inverse of a rational number is a rational number which when added to the given rational number gives '0'.

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers and $\frac{p}{q}+\frac{r}{s}=0$, then $\frac{p}{q}$ is called the additive inverse or negative of $\frac{r}{s}$ and vice-versa.

Ex. Additive inverse of $\frac{-3}{4}$ is $\frac{3}{4}$ and additive inverse of $\frac{3}{4}$ is $\frac{-3}{4}$

Ex: Find the additive inverse of negative of a rational number $\frac{-23}{11}$?

Sol. Negative of a rational number $\frac{-23}{11}$ is $-\left(\frac{-23}{11}\right)=+\frac{23}{11}$.

The additive inverse of $\frac{23}{11}$ is $-\frac{23}{11}$.

Note: The additive inverse of negative of any rational number is itself.

SUBTRACTION OF RATIONAL NUMBERS: Subtraction is the inverse process of addition.

If $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers it follows $\frac{r}{s}-\frac{p}{q}=\frac{r}{s}+\left(-\frac{p}{q}\right)$.

Ex. Subtract $\left(\frac{-2}{7}\right)$ from $\frac{3}{4}$.

Sol. $\frac{3}{4}-\left(-\frac{2}{7}\right)=\frac{3}{4}+\frac{2}{7}=\frac{29}{28}$.

Ex: Simplify: $\frac{3}{8}-\left(\frac{-2}{9}\right)+\left(\frac{-1}{36}\right)$.

Sol. $\frac{3}{8}-\left(\frac{-2}{9}\right)+\left(\frac{-1}{36}\right)=\frac{3}{8}+\frac{2}{9}-\frac{1}{36}=\frac{27+(16)+(-2)}{72}=\frac{41}{72}$

PROPERTIES OF SUBTRACTION OF RATIONAL NUMBERS:

CLOSURE PROPERTY: Let $\frac{a}{b}$ and $\frac{c}{d}$ are two unique rational numbers such that $\frac{a}{b}-\frac{c}{d}$ is also a rational number.

Ex. Let $\frac{-5}{12},\frac{-1}{4}$ are any two rational numbers,

$\left(\frac{-5}{12}-\left(\frac{-1}{4}\right)\right)=\left(\frac{-5}{12}+\frac{1}{4}\right)=\frac{-5+3}{12}=\frac{-2}{12}=\frac{-1}{6}$, here $\frac{-1}{6}$ is also a rational number.

Existence of right identity: In case of subtraction, any rational number $\frac{a}{b},\frac{a}{b}-0=\frac{a}{b}$ but $0-\frac{a}{b}=-\frac{a}{b}\left(\text{not equal to } \frac{a}{b}\right)$.

Therefore, only the right identity exists for subtraction.

Note: Properties commutative and associative do not exist for rationals under subtraction.

SYNOPSIS-2

MULTIPLICATION OF RATIONAL NUMBERS:

Let $\frac{a}{b},\frac{c}{d}$ are two rational numbers such that $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$.

Ex. a) $\frac{2}{5}\times\frac{3}{7}=\frac{6}{35}$ b) $\frac{-1}{3}\times\frac{-5}{9}=\frac{5}{27}$

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

CLOSURE PROPERTY: The product of two rational numbers is always a rational number.

i.e., $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

Ex. i) $\frac{1}{4}\times\frac{3}{2}=\frac{3}{8}\in Q$ ii) $\left(\frac{2}{5}\right)\times\left(-\frac{7}{25}\right)=-\frac{14}{125}\in Q$

iii) $\left(-\frac{7}{11}\right)\times\left(-\frac{10}{11}\right)=-\frac{70}{121}\in Q$

Commutative Property : Two rationals can be multiplied in any order.

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$

Ex. i) $\frac{1}{4}\times\frac{3}{2}=\frac{3}{2}\times\frac{1}{4}$ ii) $\frac{2}{5}\times\left(-\frac{7}{25}\right)=\left(-\frac{7}{25}\right)\times\frac{2}{5}$

iii) $\left(-\frac{7}{11}\right)\times\left(-\frac{10}{11}\right)=\left(-\frac{10}{11}\right)\times\left(-\frac{7}{11}\right)$

Note: We observe that the order of multiplication rationals does not change the sum. Then in such a case, we say that commutative property holds good under multiplication.

ASSOCIATIVE PROPERTY: When multiply three (or) more rational numbers, we can be group them in any order.

For any three rationals $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$, we have $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$

Ex. i) $\left(\frac{1}{2}\times\frac{3}{2}\right)\times\frac{5}{2}=\frac{1}{2}\times\left(\frac{3}{2}\times\frac{5}{2}\right)\Rightarrow\frac{3}{4}\times\frac{5}{2}=\frac{1}{2}\times\frac{15}{4}\Rightarrow\frac{15}{8}=\frac{15}{8}$

ii) $\left(\frac{1}{2}\times\frac{-3}{4}\right)\times\frac{5}{2}=\frac{1}{2}\times\left(\frac{-3}{4}\times\frac{5}{2}\right)\Rightarrow\left(\frac{-3}{8}\right)\times\frac{5}{2}=\left(\frac{1}{2}\times\frac{-15}{8}\right)\Rightarrow\frac{-15}{16}=\frac{-15}{16}$

EXISTENCE OF MULTIPLICATIVE IDENTITY: For any rational number $\frac{a}{b}$, we have $\frac{a}{b}\times 1=1\times\frac{a}{b}=\frac{a}{b}$. Here, '1' is called multiplicative identity for rational numbers.

Ex. i) $\frac{3}{5}\times 1=1\times\frac{3}{5}=\frac{3}{5}$ ii) $\frac{2}{11}\times 1=1\times\frac{2}{11}=\frac{2}{11}$

INVERSE PROPERTY: Every non-zero rational number $\frac{a}{b}$ has multiplicative inverse $\frac{b}{a}$, such that $\frac{a}{b}\times\frac{b}{a}=\frac{b}{a}\times\frac{a}{b}=1$

Ex. i) $\frac{2}{3}\times\frac{3}{2}=\frac{3}{2}\times\frac{2}{3}=1$. i.e., Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

ii) $-\frac{5}{3}\times-\frac{3}{5}=-\frac{3}{5}\times-\frac{5}{3}=1$. i.e., Reciprocal of $-\frac{5}{3}$ is $-\frac{3}{5}$

Note: a) $\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

b) Zero has no reciprocal.

c) Reciprocal of '1' is '1' and reciprocal of (-1) is (-1).

d) We denote the reciprocal of $\frac{a}{b}$ by $\left(\frac{a}{b}\right)^{-1}$, clearly $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$.

Distributive law of multiplication over addition: For any three rationals $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$, we have $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right)$

Ex. i) If $\frac{1}{2},\frac{2}{3}$ and $\frac{3}{4}$ are rational numbers then,

$$\frac{1}{2}\times\left(\frac{2}{3}+\frac{3}{4}\right)=\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{3}{4}\right)\Rightarrow\frac{1}{2}\times\frac{17}{12}=\frac{2}{6}+\frac{3}{8}\Rightarrow\frac{17}{24}=\frac{8+9}{24}\Rightarrow\frac{17}{24}=\frac{17}{24}$$

ii) $-\frac{3}{5},\frac{1}{3}$ and $-\frac{1}{2}$ are rational numbers, then

$$-\frac{3}{5}\times\left(\frac{1}{3}+\left(-\frac{1}{2}\right)\right)=\left(-\frac{3}{5}\times\frac{1}{3}\right)+\left(-\frac{3}{5}\times-\frac{1}{2}\right)\quad\Rightarrow\quad-\frac{3}{5}\times\left(\frac{2-3}{6}\right)=-\frac{3}{15}+\frac{3}{10}$$

$$\Rightarrow-\frac{3}{5}\times-\frac{1}{6}=-\frac{3}{15}+\frac{3}{10}\Rightarrow\frac{3}{30}=\frac{-6+9}{30}\Rightarrow\quad\frac{1}{10}=\frac{1}{10}$$

SYNOPSIS-3

DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times$ reciprocal of $\frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Ex. i) $\frac{1}{8} \div \frac{9}{2} = \frac{1}{8} \times \frac{2}{9} = \frac{1 \times 1}{4 \times 9} = \frac{1}{36}$

ii) $-2\frac{1}{3} \div 3\frac{3}{4} = \frac{-7}{3} \div \frac{15}{4} = \frac{-7}{3} \times \frac{4}{15} = \frac{-7 \times 4}{3 \times 15} = \frac{-28}{45}$

iii) $-2\frac{2}{7} \div (-8) = \frac{-16}{7} \div (-8) = \frac{-16}{7} \times \frac{-1}{8} = \frac{-2 \times -1}{7 \times 1} = \frac{2}{7}$

PROPERTIES OF DIVISION

Property-1: If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number. i.e., the set of all non-zero rational numbers is closed under division.

Ex. $\frac{27}{16} \div \frac{9}{8} = \frac{27}{16} \times \frac{8}{9} = \frac{27 \times 8}{16 \times 9} = \frac{3 \times 1}{2 \times 1} = \frac{3}{2}$ is a rational number

Property-2: For any rational number $\frac{a}{b}$, we have $\frac{a}{b} \div 1 = \frac{a}{b}$ and $\frac{a}{b} \div (-1) = -\frac{a}{b}$

Ex. $\frac{8}{21} \div 1 = \frac{8}{21}, \frac{8}{21} \div (-1) = \frac{8}{21} \div \frac{-1}{1} = \frac{8}{21} \times \frac{1}{-1} = \frac{8 \times 1}{21 \times -1} = \frac{8}{-21} = \frac{-8}{21}$

Property-3: For every non-zero rational number $\frac{a}{b}$, we have.

Ex. i) $\frac{a}{b} \div \frac{a}{b} = 1$ ii) $\frac{a}{b} \div \left(-\frac{a}{b}\right) = -1$ iii) $-\frac{a}{b} \div \frac{a}{b} = -1$

ABSOLUTE VALUE OF A RATIONAL NUMBER:

For a rational number $x, |x| = \begin{cases} x, if\ x > 0 \\ 0, if\ x = 0 \\ -x, if\ x < 0 \end{cases}$

Ex. The absolute value of $\frac{-15}{17}$ is $\left|\frac{-15}{17}\right| = \frac{15}{17}$.

Ex: Find the absolute value of $\left(\frac{-1}{3} \times \frac{7}{-3}\right)$?

Sol. $\left|\frac{-1}{3} \times \frac{7}{-3}\right| = \left|\frac{1}{3} \times \frac{7}{3}\right| = \left|\frac{7}{9}\right| = \frac{7}{9}$.

Note: i) Absolute value of a rational number is also called as Modulus value of rational.

ii) Absolute value is denoted by the symbol '$|\ |$'.

iii) Absolute value of a positive rational number remains the same.

iv) Absolute value of a negative rational is always positive.

Ex: Find absolute value of $34 - |3 \times (-4)| + |4 \times 2 - (-5)|$?

Sol. $34 - |3 \times (-4)| + |4 \times 2 - (-5)| = 34 - |-12| + |13| = 34 - 12 + 13 = 35$.

Ex: Find absolute value of $\frac{|56 - (-8)|}{|-6 - 2|} \times \left(-\frac{1}{4}\right)$?

Sol. $\frac{|56 - (-8)|}{|-6 - 2|} \times \left(-\frac{1}{4}\right) = \frac{|64|}{|-8|} \times \left(-\frac{1}{4}\right) = \frac{64}{8} \times \left(-\frac{1}{4}\right) = -2$

WORD PROBLEMS ON RATIONAL NUMBERS

(i) Lekha covered $\frac{2}{5}$ of her journey by cycle, $\frac{1}{4}$ by train and rest by car. What rational number of her journey was covered by car

Sol. Part of journey covered by cycle = $\frac{2}{5}$

Part of journey covered by train = $\frac{1}{4}$

Total part of journey covered by cycle and train = $\frac{2}{5} + \frac{1}{4} = \frac{8+5}{20} = \frac{13}{20}$

$\therefore$ Remaining part of journey = $1 - \frac{13}{20} = \frac{20-13}{20} = \frac{7}{20}$

$\therefore$ The rational number of journey covered by car is $\frac{7}{20}$

(ii). What should be added to $2\frac{1}{5}$ to get $3\frac{1}{2}$

Sol. $2\frac{1}{5}=\frac{11}{5}, \quad 3\frac{1}{2}=\frac{7}{2}$

One of the Rational number = $\frac{11}{5}$

Other rational number $=\frac{7}{2}-\frac{11}{5}=\frac{35-22}{10}=\frac{13}{10}$

1. Find the area of rectangle whose length is $2\frac{1}{5}$ cm and breadth is $2\frac{1}{7}$ cm

Sol. Given that length and breadth of rectangle are $2\frac{1}{5}$ cm, $2\frac{1}{7}$ cm respectively.

$\therefore$ Area of rectangle = length $\times$ breadth

$$=2\frac{1}{5}cm\times 2\frac{1}{7}cm=\frac{11}{5}cm\times\frac{15}{7}cm=\frac{11}{5}\times\frac{15}{7}cm^2=\frac{33}{7}cm^2$$

2. 28 pants of equal size were prepared from 35 $\frac{1}{4}$m of cloth. find the cloth used for one pant

Sol. Total cloth used = $35\frac{1}{4}m=\frac{141}{4}m$

Number of pants = 28

So, cloth is required for 1 pant = $\frac{141}{4}\div 28m$

$$=\frac{141}{4}\times\frac{1}{28}m=\frac{141}{112}m=1\frac{29}{112}m$$

3. Product of two rational numbers is $\frac{4}{7}$. If one of the rational number is $3\frac{1}{2}$ then find the other.

Sol. Product of two rational numbers = $\frac{4}{7}$

One of the rational number = $3\frac{1}{2}=\frac{7}{2}$

Other rational number = $\frac{4}{7}\div\frac{7}{2}=\frac{4}{7}\times\frac{2}{7}=\frac{8}{49}$

4. OPERATIONS & LAWS OF RATIONAL NUMBERS

WORK SHEET **LEVEL–I** **MAINS CORNER**

SINGLE CORRECT ANSWER TYPE QUESTIONS

OPERATIONS ON RATIONAL NUMBERS

1. The difference of $\frac{-2}{3}$ and $\frac{-3}{4}$ is

1) $\frac{1}{12}$ 2) $\frac{-17}{12}$ 3) $\frac{17}{12}$ 4) $\frac{-1}{12}$

2. The sum of two rational numbers is –5. If one of them is $\frac{-13}{6}$, then the other =

1) $\frac{-13}{6}$ 2) $\frac{-17}{6}$ 3) $\frac{13}{6}$ 4) $\frac{17}{6}$

3. The product of a positive rational number and a negative rational number is always………

1) negative 2) positive
3) neither positive nor negative 4) can't say

4. What number should be subtracted from $\frac{27}{13}$ so as to get $\frac{-3}{7}$?

1) $\frac{238}{91}$ 2) $\frac{338}{91}$ 3) $\frac{228}{91}$ 4) $\frac{158}{81}$

LAWS OF RATIONAL NUMBERS

5. The sum of any two rational number is always a rational number this statement indicates _____property

1) Closure 2) Commutative 3) Associative 4) Identity

6. $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$ is ________ property

1) Closure 2) Associative 3) Commutative 4) Distributive

7. Zero has no reciprocal.

1) False 2) True 3) doubtful 4) can't say

8. Additive inverse of $\frac{3}{-4}$ is

1) $\frac{3}{4}$ 2) $\frac{1}{4}$ 3) 3 4) 0

LEVEL–II

OPERATIONS ON RATIONAL NUMBERS

9. If $x=-\frac{1}{3}$ and $y=\frac{2}{7}$, then $|x+y|$ is

1) $\geq |x|+|y|$ 2) $=|x|+|y|$ 3) $<|x|+|y|$ 4) $= x+y$

10. Sum of $\frac{-4}{7}+\frac{7}{6}+\frac{2}{7}+3+\frac{-11}{6}$

1) $\frac{23}{21}$ 2) $\frac{13}{11}$ 3) $\frac{73}{21}$ 4) $\frac{43}{21}$

11. Subtract the sum of $\frac{-4}{7}$ and $\frac{5}{14}$ from the sum of $\frac{9}{14}$ and $\frac{23}{14}$

1) $\frac{5}{2}$ 2) $\frac{1}{2}$ 3) $\frac{3}{2}$ 4) $\frac{7}{2}$

12. By what number should -33/16 be divided to get -11/4?

1) 3/4 2) 3/16 3) -44/20 4) 3/20

13. If $\frac{3}{7}+x+\left(\frac{-8}{21}\right)+\frac{5}{22}=\frac{-125}{462}$, then x is

1) $\frac{6}{11}$ 2) $\frac{-5}{11}$ 3) $\frac{-6}{11}$ 4) $\frac{5}{11}$

14. If product of two numbers is $25\frac{3}{8}$. If one of them is $15\frac{9}{40}$, then other number is

1) $\frac{2}{3}$ 2) $1\frac{2}{3}$ 3) $5\frac{2}{3}$ 4) $\frac{9}{7}$

15. The area of a rectangle is $45\frac{5}{16}cm^2$. If one edge is $6\frac{1}{4}\ cm$, then the other=

1) $7\frac{1}{4}$ cm. 2) $8\frac{1}{4}$ cm. 3) $5\frac{1}{4}$ cm. 4) $3\frac{1}{4}$ cm.

16. Find absolute value of $34-|3\times(-4)|+|4\times 2-(-5)|$?

1) 45 2) 25 3) 35 4) 56

17. Find absolute value of $\frac{|56-(-8)|}{|-6-2|}\times\left(-\frac{1}{4}\right)$?

1) 4 2) -3 3) 3 4) -2

LAWS OF RATIONAL NUMBERS

18. The sum of the additive inverse and multiplicative inverse of 2 is

1) $\frac{3}{2}$ 2) $\frac{-3}{2}$ 3) $\frac{1}{2}$ 4) $\frac{-1}{2}$

19. If $\frac{p}{q}$ is the additive inverse of $\frac{r}{s}$ then $\frac{p}{q}+\frac{r}{s}=$

1) 1 2) $\frac{pr}{qs}$ 3) 0 4) $\frac{p+r}{q+s}$

20. Multiplicative inverse of $-\frac{2009}{2010}$ is

1) $-\frac{2009}{2010}$ 2) $-\frac{2010}{2009}$ 3) –1 4) 0

21. Additive inverse of $\frac{15}{16}$ is

1) 1 2) 0 3) $\frac{-15}{16}$ 4) $\frac{16}{15}$

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

22. The speed of car is $54\frac{1}{2}$ km per hour. What is the distance travelled in $\frac{7}{2}$ hours and $\frac{35}{2}$ minutes?

1) $\frac{4929}{48}$ km 2) $\frac{9972}{48}$ km 3) $\frac{14279}{48}$ km 4) $\frac{2479}{24}$ km

23. If $15\frac{2}{3}\times 3\frac{1}{6}+6\frac{1}{3}=11\frac{7}{8}+x$, then the value of x is

1) $39\frac{5}{9}$ 2) $137\frac{4}{9}$ 3) $29\frac{7}{9}$ 4) $44\frac{5}{72}$

24. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)........\left(1-\frac{1}{n}\right)$ is

1) 1 2) $\left(1-\frac{1}{n}\right)^2$ 3) $\frac{1}{n}$ 4) None of these

LEVEL–IV

STATEMENT TYPE QUESTIONS

25. Statement I: The additive identity in rational number is 0
Statement II: The additive inverse of a is –a
1) Both Statements are true. 2) Both Statements are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

26. Statement I: If x, y, z be rational numbers such that x > y and y > z, then x > z
Statement II: The sum of two rational numbers is always greater than third rational numbers.

1) Both Statements are true. 2) Both Statements are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

INTEGER TYPE QUESTIONS

27. Verify the distributive property $x\times(y+z)=xy+xz$ where $x=\frac{-4}{3}, y=\frac{1}{2}, z=\frac{-7}{5}$ and write the unit digit denominator of $(x+y)\times z$

28. Find the sum of numerator and denominator of reciprocal of rational number $\frac{2}{5}+\frac{5}{4}$.

29. The product of two numbers is $\frac{49}{56}$. If one of them is 9/7, then denominator of other number is _______.

MULTI CORRECT ANSWER TYPE QUESTIONS

30. Which of these are not reciprocals of rational numbers lying between 1 and 4?

1) $\frac{3}{4}$ 2) $\frac{4}{3}$ 3) $\frac{5}{17}$ 4) $\frac{2}{9}$

31. The sum of two rational numbers is -3. If one of the numbers is $-\frac{10}{3}$, the other number is

1) $\frac{-13}{3}$ 2) $\frac{-19}{3}$ 3) $\frac{1}{3}$ 4) $\left(\frac{81}{27}\right)^{-1}$

32. The product of two numbers is $-\frac{16}{35}$. If one of the numbers is $-\frac{15}{14}$, then the other.

1) $-\frac{2}{5}$ 2) $\frac{-32}{-75}$ 3) $\frac{32}{75}$ 4) $-\frac{8}{3}$

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE-1

If $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)$ $=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$

33. $\frac{2}{3}\times\frac{-7}{10}+\frac{-2}{3}\times\frac{8}{9}=x\left[\frac{-7}{10}+y\right]$ then x, y are equal to

1) $\frac{2}{3},\frac{8}{9}$ 2) $\frac{-2}{3},\frac{-8}{9}$ 3) $\frac{2}{3},\frac{-8}{-9}$ 4) $\frac{+2}{3},\frac{-8}{9}$

34. Name the property used above?

1) Commutativity of multiplication over addition

2) Commutativity of addition over multiplication

3) Distributivity of multiplication over addition

4) Distributivity of addition over multiplication

35. $\frac{2}{5}\times\frac{-8}{9}+p\times\frac{5}{9}=\frac{2}{5}\times[q+r]$ then p, q, r are equal to

1) $\frac{2}{5},\frac{-8}{9},\frac{5}{9}$ 2) $\frac{2}{5},\frac{8}{9},\frac{-5}{9}$ 3) $\frac{-2}{5},\frac{-8}{9},\frac{-5}{9}$ 4) $\frac{-2}{5},\frac{-8}{9},\frac{5}{9}$

PASSAGE-2

From a starting point A, Rahul walks $\frac{3}{4}$ km towards east and then $\frac{6}{7}$ km towards west to reach point C.

36. Where will he be now from the starting point A?

1) $\frac{9}{28}$ towards west 2) $\frac{3}{28}$ towards west

3) $\frac{3}{28}$ towards East 4) $\frac{9}{28}$ towards east

37. How much total distance Rahul walks to reach point C?

1) $\frac{45}{28}$ km 2) $\frac{43}{28}$ km 3) $\frac{47}{28}$ km 4) $\frac{49}{28}$ km

38. How much more distance he walked towards east than west?

1) $\frac{-9}{28}$ 2) $\frac{3}{28}$ 3) $\frac{9}{28}$ 4) $\frac{-3}{28}$

MATRIX MATCH TYPE QUESTIONS

39. Match the following

	Column-I		Column-II
a	Associative law	p	If a & b rational number, then a + b is rational
b	Commutative law	q	If a & b are rational numbers, then $a+b=b+a$
c	Distributive law	r	If a, b & c are rational numbers, then $a+(b+c)=(a+b)+c$
d	Closure law	s	If a, b & c are rational numbers, then $a\times(b+c)=ab+ac$
		t	$a\times b=1$